

Fracturas en Distribuciones autogravitantes cargadas

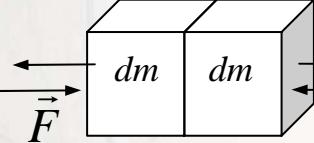
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Fractura



$$a^\alpha = \left[-R^\alpha{}_{\beta\gamma\delta} u^\beta u^\delta + h^\alpha{}_\beta \left(\frac{du^\beta}{ds} \right)_{;\gamma} - \frac{du^\beta}{ds} \frac{du_\gamma}{ds} \right] \delta_\perp x^\gamma \quad (1)$$

La ecuación de Raychaudhuri gobierna la evolución temporal de la expansión

$$\frac{d\Theta}{ds} = -R_{\mu\nu} u^\mu u^\nu + \left(\frac{du^\mu}{ds} \right)_{;\mu} + 2(\Omega^2 - \sigma^2) - \frac{1}{3} \Theta^2 \quad (2)$$

Podemos relacionar la fuerza radial con (2) mediante

$$R = -\frac{e^\lambda (\rho + P)}{r^2 e^{\nu/2}} \int_0^a r^2 e^{\nu/2} \frac{d\Theta}{ds} dr \quad (3)$$

es equivalente a observar el signo de la fuerza radial sobre cada elemento de fluido después de la perturbación

$$\text{sign}[R] \Leftrightarrow \text{sign}[\Theta]$$

Herrera *Phys. Lett A*, **165**, 296 (1992); Di Prisco et al *Phys. Lett A*, **195**, 23 (1994)
 Di Prisco, Herrera and Varela *GRG* **29**, 10 (1997)

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Fracturas en Distribuciones
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Para un fluido anisótropo descrito con la métrica

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (4)$$

La ecuación de equilibrio hidrostático

$$R \equiv \frac{dP}{dr} + (\rho + P) \left(\frac{m + 4\pi r^3 P}{r(r - 2m)} \right) - \frac{2}{r} (P_T - P) \quad (5)$$

Bajo perturbaciones de densidad y anisotropía

$$\begin{aligned} \rho + \delta\rho &\Rightarrow \begin{cases} P_r(\rho + \delta\rho, r) \approx P_r(\rho, r) + \frac{\partial P_r}{\partial \rho} \delta\rho \\ m(\rho + \delta\rho, r) = 4\pi \int_0^r (\rho + \delta\rho) r^{-2} dr \approx m(\rho, r) + \frac{4\pi}{3} r^3 \delta\rho \end{cases} \\ \Delta + \delta\Delta \end{aligned} \quad (6)$$

Fracturas en Distribuciones
autogravitantes cargadas

$$R = R(\rho_0 + \delta\rho, P_r, \Delta_0 + \delta\Delta) \equiv \underbrace{R_0(\rho_0, P_r, \Delta_0)}_{=0} + \underbrace{\frac{\partial R}{\partial \rho} \delta\rho + \frac{\partial R}{\partial P_r} \delta P_r + \frac{\partial R}{\partial m} \delta m + \frac{\partial R}{\partial \Delta} \delta\Delta}_{\tilde{R}} \quad (7)$$

$$\tilde{R} = \delta P \left[\left(2 \frac{\partial R}{\partial \rho} + \frac{4\pi}{3} r^3 \frac{\partial R}{\partial m} \right) - \frac{2}{r} \frac{\delta\Delta}{\delta\rho} \right] \quad (8)$$

Donde

$$\frac{\partial R}{\partial \rho} = \frac{m + 4\pi P_r r^3}{r(r - 2m)} \geq 0 \quad \text{y} \quad \frac{\partial R}{\partial m} = \frac{(\rho + P)(1 + 8\pi P_r r^2)}{(r - 2m)^2} \geq 0 \quad (9)$$

$$\frac{\delta\Delta}{\delta\rho} \approx \frac{\delta(P_T - P_r)}{\delta\rho} \approx \frac{\delta P_T}{\delta\rho} - \frac{\delta P_r}{\delta\rho} \approx v_{sT}^2 - v_{sr}^2 \quad (10)$$

Donde v_{sT} y v_{sr} representan la velocidad del sonido tangencial y radial respectivamente

Es claro que debido a $0 \leq v_{sr}^2 \leq 1$ y a $0 \leq v_{sT}^2 \leq 1$ tendremos $|v_{sT}^2 - v_{sr}^2| \leq 1$, y en consecuencia

$$-1 \leq v_{sT}^2 - v_{sr}^2 \leq 1 \Rightarrow \begin{cases} -1 \leq v_{sT}^2 - v_{sr}^2 \leq 0 & \text{Potencialmente estable} \\ 0 \leq v_{sT}^2 - v_{sr}^2 \leq 1 & \text{Potencialmente inestable} \end{cases} \quad (11)$$

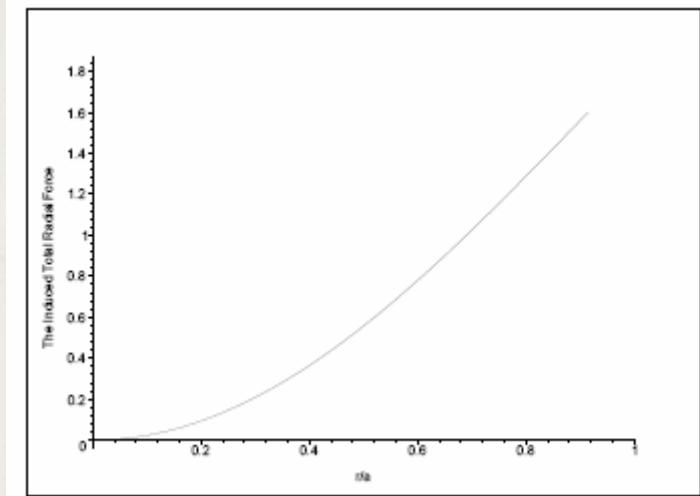
Perturbaciones y fracturas para configuraciones anisótropas

- Stewart no local 1 $\rightarrow \rho = \frac{1}{8\pi r^2} \frac{(e^{2Kr} - 1)(e^{4Kr} + 8Kre^{2Kr} - 1)}{(e^{2Kr} + 1)^3}$
(*J Phys. A.* **15**, 2419 (1982))

$$\bullet \text{ Stewart no local 2 } \rightarrow \rho = \frac{1}{8\pi r^2} \left[1 - \frac{\sin(2Kr)}{Kr} + \frac{\sin^2(Kr)}{K^2 r^2} \right] \quad (13)$$

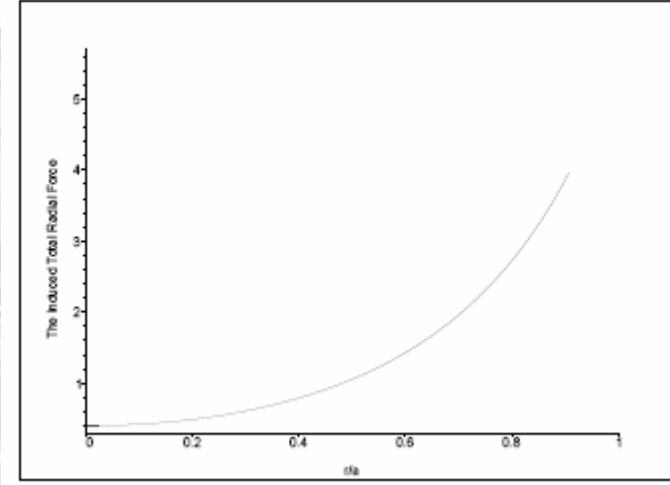
$$\bullet \text{ Gokhroo \& Mehra } \rho = \rho_c \left(1 - \frac{Kr^2}{a^2} \right), P_r = \frac{\rho_c}{j} \left(1 - \frac{2Mr^2}{a^3} \left[\frac{5 - \frac{3Kr^2}{a^2}}{5 - 3K} \right] \right) \left(1 - \frac{r^2}{a^2} \right)^n \quad (14)$$

Fracturas en Distribuciones autogravitantes cargadas



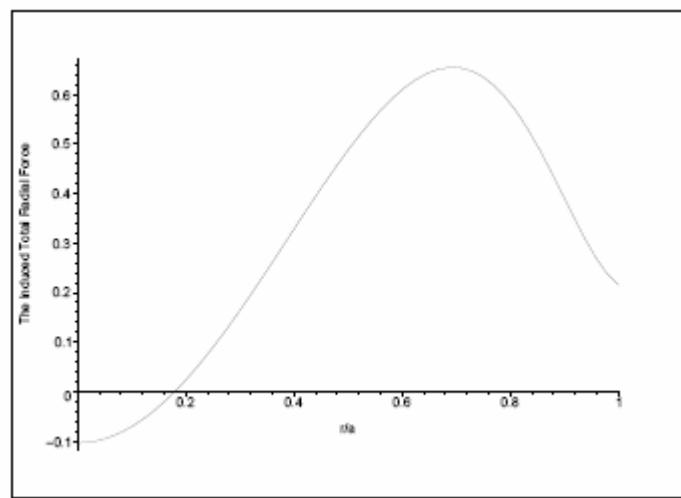
Stewart 1

(*J Phys. A.* **15**, 2419 (1982))



Stewart 2

(*J Phys. A.* **15**, 2419 (1982))



Gokhroo & Mehra
(*GRG*, **26**, 75 (1994))

Fracturas en Distribuciones autogravitantes cargadas

Fracturas cargadas

$$ds^2 = \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right) dt^2 - \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) \quad (15)$$

$$R \equiv \frac{dP_r}{dr} - \frac{qq'}{4\pi r^4} + (\rho + P_r) \left(4\pi r P_r + \frac{m}{r^2} - \frac{q^2}{r^3} \right) \left(1 - \frac{2m}{r} + \frac{q^2}{r^2} \right)^{-1} - \frac{2}{r} \Delta \quad (16)$$

Para adimensionalizar R renombramos a los parámetros

$$\begin{aligned} P &= P_c \tilde{P}, & m &= M \tilde{m}, & \tau &= a^2 P_c, & \Lambda &= \frac{M}{a}, \\ q &= Q \tilde{q}, & r &= a \eta, & Z &= \frac{\rho_c}{P_c}, & \tilde{Q} &= \frac{Q}{M}, \\ \rho &= \rho_c \tilde{\rho}, & \Delta &= P_c \tilde{\Delta}, & & & & \end{aligned} \quad (17)$$

Además si sustituimos $\chi = \tilde{q}^2$

$$\left. \begin{aligned} \chi' &= 2qq' \Rightarrow qq' = \frac{\chi'}{2} \\ \xi &= qq' = \frac{1}{2} \frac{d\chi}{dr} \end{aligned} \right\} \quad \xi + \delta\xi \rightarrow \chi + \delta\chi \quad (18)$$

Fracturas en Distribuciones autogravitantes cargadas

La ecuación de equilibrio hidroestático adimensional es

$$R \equiv \frac{d\tilde{P}_r}{d\eta} - \frac{\xi}{8\pi T \eta^4} - \left(Z\tilde{\rho} + \tilde{P}_r \right) \frac{\left(4\pi\eta T \tilde{P}_r + \frac{\Lambda\tilde{m}}{\eta^2} - \Lambda\tilde{\chi} \right)}{\left(1 - 2\Lambda\tilde{m} + \Lambda^2\tilde{\chi} \right)} - \frac{2}{\eta}\tilde{\Delta} \quad (19)$$

Podemos perturbar carga

$$R = R(\rho_0, P_r, \Delta_0, \chi_0 + \delta\chi, \xi_0 + \delta\xi) \equiv \underbrace{R_0(\rho_0, P_r, \Delta_0, \tilde{\chi}_0 = 0, \xi_0 = 0)}_{=0} + \underbrace{\frac{\partial R}{\partial \rho} \Big|_{\substack{\tilde{\chi}=0 \\ \xi=0}} \delta\rho + \frac{\partial R}{\partial m} \Big|_{\substack{\tilde{\chi}=0 \\ \xi=0}} \delta m + \frac{\partial R}{\partial \Delta} \Big|_{\substack{\tilde{\chi}=0 \\ \xi=0}} \delta\Delta + \boxed{\frac{\partial R}{\partial \xi} \Big|_{\substack{\tilde{\chi}=0 \\ \xi=0}} \delta\xi + \frac{\partial R}{\partial \tilde{\chi}} \Big|_{\substack{\tilde{\chi}=0 \\ \xi=0}} \delta\tilde{\chi}}}_{\tilde{R}} \quad (20)$$

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$$R \equiv \frac{d\tilde{P}_r}{d\eta} - \frac{\xi}{8\pi T \eta^4} - (Z\tilde{\rho} + \tilde{P}_r) \frac{\left(4\pi\eta T \tilde{P}_r + \frac{\Lambda\tilde{m}}{\eta^2} - \Lambda\tilde{\chi}\right)}{(1 - 2\Lambda\tilde{m} + \Lambda^2\tilde{\chi})} - \frac{2\tilde{\Delta}}{\eta} \quad (19)$$

Podemos perturbar carga y densidad

$$R = R(\rho_0 + \delta\rho, P_r, \Delta_0, \chi_0 + \delta\chi, \xi_0 + \delta\xi) \equiv \underbrace{R_0(\rho_0, P_r, \Delta_0, \tilde{\chi}_0 = 0, \xi_0 = 0) +}_{=0} + \underbrace{\left[\frac{\partial R}{\partial \rho} \Bigg|_{\begin{subarray}{l} \tilde{\chi}=0 \\ \xi=0 \end{subarray}} \delta\rho + \frac{\partial R}{\partial m} \Bigg|_{\begin{subarray}{l} \tilde{\chi}=0 \\ \xi=0 \end{subarray}} \delta m + \frac{\partial R}{\partial \Delta} \Bigg|_{\begin{subarray}{l} \tilde{\chi}=0 \\ \xi=0 \end{subarray}} \delta\Delta + \frac{\partial R}{\partial \xi} \Bigg|_{\begin{subarray}{l} \tilde{\chi}=0 \\ \xi=0 \end{subarray}} \delta\xi + \frac{\partial R}{\partial \tilde{\chi}} \Bigg|_{\begin{subarray}{l} \tilde{\chi}=0 \\ \xi=0 \end{subarray}} \delta\tilde{\chi} \right]}_{\tilde{R}} \quad (20)$$

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Podemos perturbar carga y anisotropía

$$R = R(\rho_0 + \delta\rho, P_r, \Delta_0 + \delta\Delta, \chi_0 + \delta\chi, \xi_0 + \delta\xi) \equiv \underbrace{R_0(\rho_0, P_r, \Delta_0, \tilde{\chi}_0 = 0, \xi_0 = 0)}_{=0} + \underbrace{\frac{\partial R}{\partial \rho} \Big|_{\substack{\tilde{\chi}=0 \\ \xi=0}} \delta\rho + \frac{\partial R}{\partial m} \Big|_{\substack{\tilde{\chi}=0 \\ \xi=0}} \delta m + \frac{\partial R}{\partial \Delta} \Big|_{\substack{\tilde{\chi}=0 \\ \xi=0}} \delta\Delta + \frac{\partial R}{\partial \xi} \Big|_{\substack{\tilde{\chi}=0 \\ \xi=0}} \delta\xi + \frac{\partial R}{\partial \tilde{\chi}} \Big|_{\substack{\tilde{\chi}=0 \\ \xi=0}} \delta\tilde{\chi}}_{\tilde{R}} \quad (20)$$

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La ecuación de equilibrio hidroestático adimensional es

$$R \equiv \frac{d\tilde{P}_r}{d\eta} - \frac{\xi}{8\pi T \eta^4} - \left(Z\tilde{\rho} + \tilde{P}_r \right) \frac{\left(4\pi\eta T \tilde{P}_r + \frac{\Lambda\tilde{m}}{\eta^2} - \Lambda\tilde{\chi} \right)}{\left(1 - 2\Lambda\tilde{m} + \Lambda^2\tilde{\chi} \right)} - \frac{2\tilde{\Delta}}{\eta} \quad (19)$$

Podemos perturbar

$$R = R(\rho_0 + \delta\rho, P_r, \Delta_0 + \delta\Delta, \chi_0 + \delta\chi, \xi_0 + \delta\xi) \equiv \underbrace{R_0(\rho_0, P_r, \Delta_0, \tilde{\chi}_0 = 0, \xi_0 = 0)}_{=0} + \underbrace{\left[\frac{\partial R}{\partial \rho} \Big|_{\substack{\tilde{\chi}=0 \\ \xi=0}} \delta\rho + \frac{\partial R}{\partial m} \Big|_{\substack{\tilde{\chi}=0 \\ \xi=0}} \delta m + \frac{\partial R}{\partial \Delta} \Big|_{\substack{\tilde{\chi}=0 \\ \xi=0}} \delta\Delta + \frac{\partial R}{\partial \xi} \Big|_{\substack{\tilde{\chi}=0 \\ \xi=0}} \delta\xi + \frac{\partial R}{\partial \tilde{\chi}} \Big|_{\substack{\tilde{\chi}=0 \\ \xi=0}} \delta\tilde{\chi} \right]}_{\tilde{R}} \quad (20)$$

Caso 1: Perturbando carga

- $\xi + \delta\xi \rightarrow \int (\xi + \delta\xi) dr \approx \tilde{\chi}(\xi, r) + \frac{M}{a\eta} \delta\tilde{\chi}$

$$R = R(\rho, P_r, \Delta, \chi_0 + \delta\chi, \xi_0 + \delta\xi) \equiv \underbrace{R_0(\rho_0, P_r, \Delta_0, \tilde{\chi}_0 = 0, \xi_0 = 0)}_{=0} + \quad (21)$$

$$+ \underbrace{\left. \frac{\partial R}{\partial \xi} \right|_{\begin{subarray}{l} \tilde{\chi}=0 \\ \xi=0 \end{subarray}} \delta\xi + \left. \frac{\partial R}{\partial \tilde{\chi}} \right|_{\begin{subarray}{l} \tilde{\chi}=0 \\ \xi=0 \end{subarray}} \delta\tilde{\chi}}_{\tilde{R}}$$

R perturbada:

$$\tilde{R} = \left(\frac{(Z\tilde{\rho} + \tilde{P})\Lambda^2}{1 - 2\Lambda\tilde{m}} + \frac{(Z\tilde{\rho} + \tilde{P}) \left(4\pi T \eta \tilde{P} + \frac{\Lambda\tilde{m}}{\eta^2} \right) \Lambda^2}{(1 - 2\Lambda\tilde{m})^2} - \frac{1}{8} \frac{\Lambda}{T\eta^5} \right) \delta\tilde{\chi} \quad (22)$$

Fracturas en Distribuciones autogravitantes cargadas

Caso 2: Perturbando carga y densidad

- $\xi + \delta\xi \rightarrow \int (\xi + \delta\xi) dr \approx \tilde{\chi}(\xi, r) + \frac{M}{a\eta} \delta\tilde{\chi}$ (22)
- $\rho + \delta\rho \Rightarrow m(\rho + \delta\rho, r) = 4\pi \int_0^r (\rho + \delta\rho) r^2 dr \approx m(\rho, r) + \frac{4\pi}{3} r^3 \delta\rho$

R perturbada:

$$\tilde{R} = \frac{(Z\tilde{\rho} + \tilde{P})\Lambda}{1 - 2\Lambda\tilde{m}} \left[\Lambda \left(1 + \frac{\left(4\pi T \eta \tilde{P} + \frac{\Lambda \tilde{m}}{\eta^2} \right)}{(1 - 2\Lambda\tilde{m})} - \frac{1}{8} \frac{(1 - 2\Lambda\tilde{m})}{T \eta^5 \Lambda (Z\tilde{\rho} + \tilde{P})} \right) \delta\tilde{\chi} - \right.$$

$$\left. \frac{1}{\eta^2} \left(1 + \frac{3}{4} \frac{Z \left(4\pi T \eta \tilde{P} + \frac{\Lambda \tilde{m}}{\eta^2} \right)}{\pi \eta T^2 (Z\tilde{\rho} + \tilde{P})} + \frac{2\eta^2 \left(4\pi T \eta \tilde{P} + \frac{\Lambda \tilde{m}}{\eta^2} \right)}{(1 - 2\Lambda\tilde{m})} \right) \delta\tilde{m} \right] \quad (23)$$

Fracturas en Distribuciones autogravitantes cargadas

Caso 3: Perturbando carga y anisotropía

- $\xi + \delta\xi \rightarrow \int (\xi + \delta\xi) dr \approx \tilde{\chi}(\xi, r) + \frac{M}{a\eta} \delta\tilde{\chi}$
- $\tilde{\Delta} + \delta\tilde{\Delta}$

R perturbada:

$$\tilde{R} = \frac{(Z\tilde{\rho} + \tilde{P})\Lambda}{1 - 2\Lambda\tilde{m}} \left(1 + \frac{\left(4\pi T\eta\tilde{P} + \frac{\Lambda\tilde{m}}{\eta^2} \right)}{(1 - 2\Lambda\tilde{m})} - \frac{1}{8} \frac{(1 - 2\Lambda\tilde{m})}{T\eta^5\Lambda(Z\tilde{\rho} + \tilde{P})} \right) \delta\tilde{\chi} - \frac{2\delta\tilde{\Delta}}{\eta} \Lambda \quad (25)$$

Fracturas en Distribuciones
autogravitantes cargadas

Caso 4: Perturbando carga, densidad y anisotropía

- $\xi + \delta\xi \rightarrow \int (\xi + \delta\xi) dr \approx \tilde{\chi}(\xi, r) + \frac{M}{a\eta} \delta\tilde{\chi}$ (26)
- $\rho + \delta\rho \Rightarrow m(\rho + \delta\rho, r) = 4\pi \int_0^r (\rho + \delta\rho) r^2 dr \approx m(\rho, r) + \frac{4\pi}{3} r^3 \delta\rho$
- $\tilde{\Delta} + \delta\tilde{\Delta}$

$$R \text{ perturbada: } \tilde{R} = \frac{(Z\tilde{\rho} + \tilde{P})\Lambda}{1 - 2\Lambda\tilde{m}} \left[\Lambda \left(1 + \frac{\left(4\pi T \eta \tilde{P} + \frac{\Lambda\tilde{m}}{\eta^2} \right)}{(1 - 2\Lambda\tilde{m})} - \frac{1}{8} \frac{(1 - 2\Lambda\tilde{m})}{T\eta^5 \Lambda (Z\tilde{\rho} + \tilde{P})} \right) \delta\tilde{\chi} - \right. \\ \left. \frac{1}{\eta^2} \left(1 + \frac{3}{4} \frac{Z \left(4\pi T \eta \tilde{P} + \frac{\Lambda\tilde{m}}{\eta^2} \right)}{\pi \eta T^2 (Z\tilde{\rho} + \tilde{P})} + \frac{2\eta^2 \left(4\pi T \eta \tilde{P} + \frac{\Lambda\tilde{m}}{\eta^2} \right)}{(1 - 2\Lambda\tilde{m})} \right) \delta\tilde{m} \right] - \frac{2\delta\tilde{\Delta}}{\eta} \quad (27)$$

Fracturas en Distribuciones autogravitantes cargadas

Resultados

Caso 1: Perturbando carga $\delta\tilde{\chi} = 1.10^{-8}$

- Stewart no local 1

(*J Phys. A.* **15**, 2419 (1982))

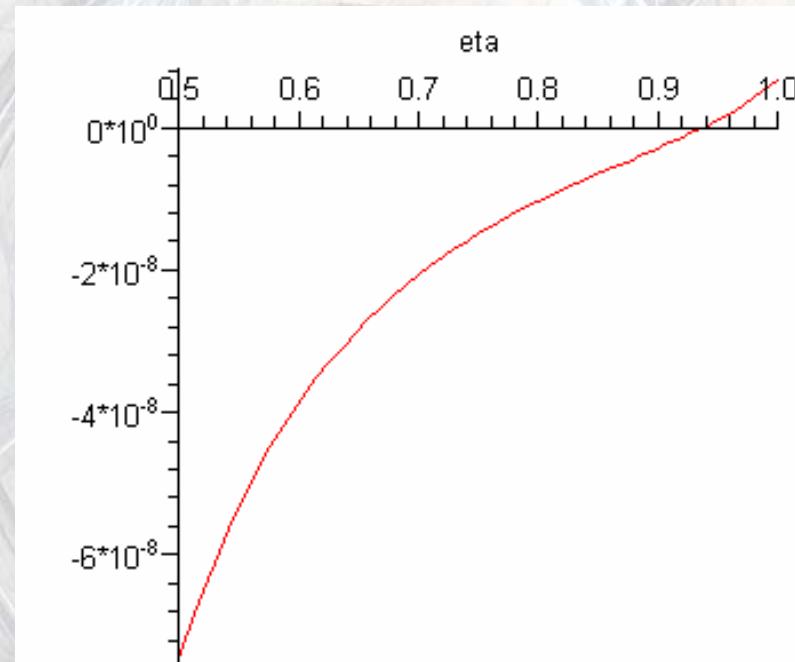
$$\rho = \frac{1}{8\pi r^2} \frac{(e^{2Kr} - 1)(e^{4Kr} + 8Kre^{2Kr} - 1)}{(e^{2Kr} + 1)^3} \quad (28)$$

↓

$$P_r = \frac{1}{8\pi r^2} \frac{(1 - e^{2Kr})(e^{4Kr} - 8Kre^{2Kr} - 1)}{(e^{2Kr} + 1)^3} \quad (29)$$

↓

$$P_T = \frac{2K^2 e^{4Kr}}{\pi(e^{2Kr} + 1)^4} \quad (30)$$



Fracturas en Distribuciones
autogravitantes cargadas

Resultados

Caso 2: Perturbando carga y densidad

- Stewart no local 1

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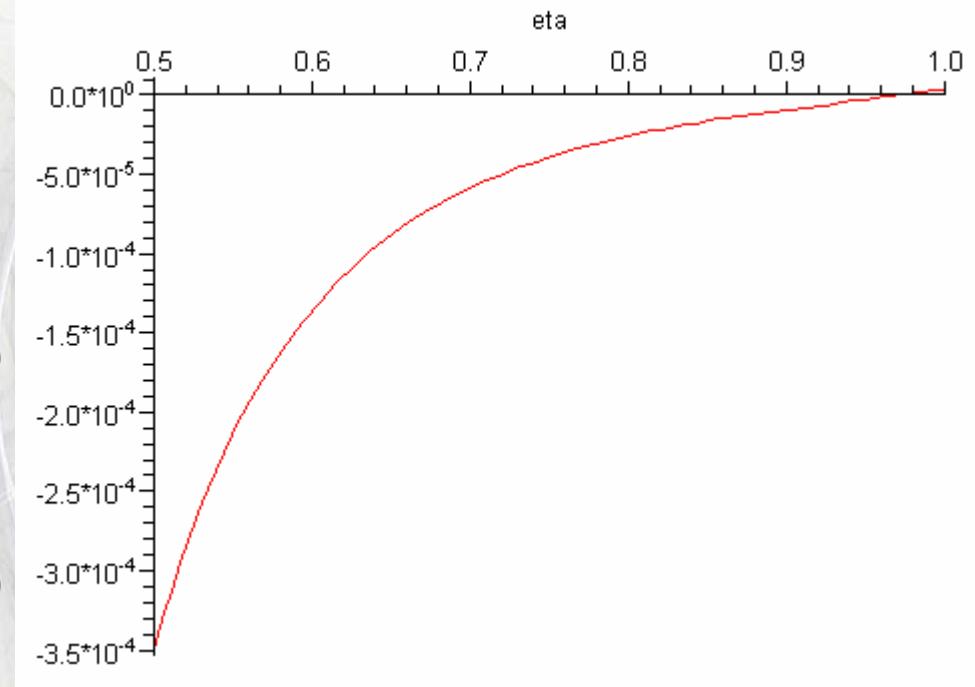
$$\rho = \frac{1}{8\pi r^2} \frac{(e^{2Kr} - 1)(e^{4Kr} + 8Kre^{2Kr} - 1)}{(e^{2Kr} + 1)^3} \quad (28)$$



$$P_r = \frac{1}{8\pi r^2} \frac{(1 - e^{2Kr})(e^{4Kr} - 8Kre^{2Kr} - 1)}{(e^{2Kr} + 1)^3} \quad (29)$$



$$P_T = \frac{2K^2 e^{4Kr}}{\pi (e^{2Kr} + 1)^4} \quad (30)$$



$$\delta \tilde{m} = 1.10^{-8}, \quad \delta \tilde{\chi} = 1.10^{-5}, \quad \frac{\delta q}{\delta m} = 3,16 \cdot 10^5$$

Fracturas en Distribuciones autogravitantes cargadas

Resultados

Caso 2: Perturbando carga y densidad

- Stewart no local 1

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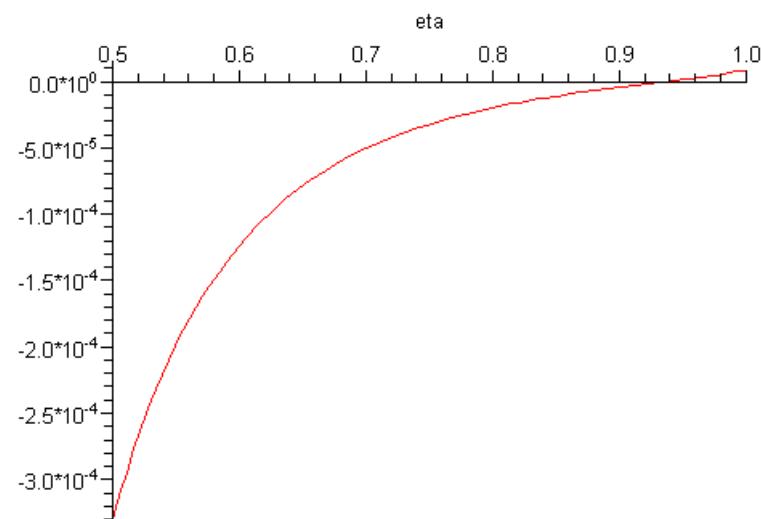
$$\rho = \frac{1}{8\pi r^2} \frac{(e^{2Kr} - 1)(e^{4Kr} + 8Kre^{2Kr} - 1)}{(e^{2Kr} + 1)^3} \quad (28)$$



$$P_r = \frac{1}{8\pi r^2} \frac{(1 - e^{2Kr})(e^{4Kr} - 8Kre^{2Kr} - 1)}{(e^{2Kr} + 1)^3} \quad (29)$$



$$P_T = \frac{2K^2 e^{4Kr}}{\pi(e^{2Kr} + 1)^4} \quad (30)$$



$$\delta \tilde{m} = -1.10^{-8}, \delta \tilde{\chi} = 1.10^{-5}, \frac{\delta q}{\delta m} = 3,16 \cdot 10^5$$

Fracturas en Distribuciones autogravitantes cargadas

Conclusiones

- Se encontró que las perturbaciones en la carga inducen fractura en modelos de esferas anisótropas autogravitantes.
- Las perturbaciones de carga que inducen fracturas pueden ser:

$$\delta\xi \neq 0, \quad \delta\tilde{\chi} \neq 0, \quad \delta\rho = 0, \quad \delta m = 0, \quad \delta\Delta = 0$$

y $\delta\xi \neq 0, \quad \delta\tilde{\chi} \neq 0, \quad \delta\rho \neq 0, \quad \delta m \neq 0, \quad \delta\Delta = 0$

- Todas las perturbaciones de carga se hicieron sobre modelos neutros anisótropos
- Se encontró que todas las perturbaciones

$$\delta\xi \neq 0, \quad \delta\tilde{\chi} \neq 0, \quad \delta\rho = 0, \quad \delta m = 0, \quad \delta\Delta = 0$$

fracturaron para $\frac{\sqrt{\delta\tilde{\chi}}}{\delta m} = \frac{\delta q}{\delta m} = 1.10^5$

Fracturas en Distribuciones
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