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Mathematical modeling in decision making process under conditions of uncertainty in human resources training and development

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Abstract

The aim of this paper is the applying, in a particular case of human resources training and development, of some mathematical models of decision making under conditions of uncertainty. The models are also known in other applications, from other fields. In this article, we wanted to show that they can be also applied in human resources training and development, which represents an original contribution in this field. In uncertainty situations, the decision-maker can not evaluate the apparition probability of the different stages of nature, since he does not have enough information and the variables are partially controllable. In such situations, the decision-maker can resort, for choosing the decisional variants, to different models (rules, techniques and criterions), that are part of the decisional theory, such as: max-min (Abraham Wald's); max-max (optimist); pessimist optimist (optimal); min-max (the criterion of reducing the regret; Savage's); equiprobability's or insufficient reason's (Laplace's). Each model has a different vision of the manifestation probability of the future events and they consequences.

key words. Decision; attitude towards uncertainty; criterions of decisions in conditions of uncertainty.

AMS subject classifications. 91B30, 91B99, 91D99

1 Introduction

Uncertainty is generated by the lack of information in all the phases of elaborating the decision (defining the problem, setting the objectives, identifying the alternatives of action, estimating the states of nature and their probabilities of apparition using the selected action). In the meantime, it is also a consequence of the lack of former experience that allows the estimation of the probabilities of apparition of the significant states of nature.

In conditions of uncertainty more states of nature may appear, thus the decision-maker can not evaluate their probability of apparition. Therefore, no matter what alternative action we will choose, it will lead to consequences with unknown probabilities of apparition. In these situations, foreign and Romanian authors underlined the fact that there are more rules available, methods or criterions that may be used. Each criterion offers a different vision of the probability of manifestation of the future events.

The decision-makers are unique. They have different perceptions upon the probability of apparition of the nature states, but also different attitudes towards risk. All of these will influence future decisions, in one way or another. Therefore, it can not be said that a criterion of decision, in conditions of uncertainty, is better or worse than another criterion. Each criterion has a specific utility for a decision-maker or another, as long as different persons mostly have different opinions on solving a problem.

2 The Main Result

Within the decision theory, solving of the situations of total uncertainty is approached with the help of some criterions (methods, models) related to the decision-maker's attitude.[6,7,8] (Table 1)

Table 1. Model of decision matrix for the uncertain situation

Decision variants	Nature states			
	S_1	S_2	$\ldots S_j \ldots$	S_n
V_1	C_{11}	C_{12}	$\ldots C_{1j} \ldots$	C_{1n}
V_2	C_{21}	C_{22}	$\ldots C_{2j} \ldots$	C_{2n}
. V_i .	. C_{i1} .	. C_{i2} .	. C_{ij} .	. C_{in} .
V_m	C_{m1}	C_{m2}	C_{mj}	C_{mn}

Where:

$V_i, \ldots, V_m$ = and $i = 1, \ldots, m$	forms the columns of the table and represents the set of variants – independent variables of decision – from which is chosen the optimal one.
$S_i, \ldots, S_n$ = and $i = 1, \ldots, n$	forms the set of nature states, the parameters respectively, the independent uncontrollable variables, identified and placed in the columns of the table.
Cij =	the result (the consequences) recorded as result of the apparition probability „j“ on the alternative „i“

For better analyzing these decisions in the domain of human resources development, the following example is worth mentioning. We assume that the company "X Ltd." is a computer manufacturer whose target is obtaining competitive advantage by improving actual computer characteristics and even by producing new types of such products within three years [14].

For the board it is obvious that this requires, among other managerial decisions, the existence of some competent human resources to achieve the management objectives. In consequence, relies on the HR department for consultancy and proposals on strategies (variants) of forming the existent human resource of the company for reaching the goal. The specialists of the HR department could not forecast the level of the human resources competencies for the next three years, so that the company could obtain the intended target. For this reason, the HR department and the HR management considered three situations for the state of competencies in realizing of the set performances: high increase, average increase, low increase.

There were also proposed three ways of forming the human resources:

1. stage for specialization and skills acquiring needed for major changes in the computer production;

2. developing program skills of solving the current needs imposed by the development of the computer production;

3. programs for acquiring new knowledge in the domain.

Analyzing the information above, the board estimated and concluded upon the efficiencies as showed in Table 2.

Table 2. Decision matrix in uncertainty conditions %

Nature states / Alternatives	S_1 High increase	S_2 Average increase	S_3 Low increase
A_1 – training programs	15	3	- 2
A_2 – programs for developing skills	12	6	3
A_3 – programs for acquiring new knowledge	6.5	6.5	6.5

Therefore, according to the above table:

- **if a big increase of competencies appears**: the training programs will bring profits of 15%; the programs of skills training will bring profits of 12%; the programs of new knowledge acquiring of 6.5%;

- **if there is a medium increase of competencies**: the training programs will bring profits of 3%; the programs of skills training will bring profits of 6%; the programs of new knowledge acquiring of 6.5%;

- **if there is a low increase of competencies**: the training programs will bring profits of - 2%; the programs of skills training will bring profits of 3%; the programs of new knowledge acquiring of 6.5%.

In terms of certainty, it is easy to analyze the situation the decision-maker indicates the best payment, the consequence from the column respectively. Specifically examining the information from the decision matrix of the example, if the decision-maker knows that the appearing nature state is S_1, he will choose the alternative A_1 because it has the highest percentage value, namely (15%). Further and following the same logic, if nature state S_2 appears, he will choose the alternative A_3 because it has the highest percentage value, namely (6.5%), and lastly, if nature state S_3 appears, the decision-maker will choose A_3 again.

This type of matrix is used when the number of the candidate alternatives is limited and usually small. This matrix approved a typical managerial problem when the decision-maker had to realize an optimal choosing between three courses of action presented in the matrix of the results. The choosing was made under certainty and then, in the following paragraphs, the same matrix will be used to see how the decision-maker will choose the optimal option in condition of uncertainty.

Maxi-min criterion (Abraham Walt's criterion)

This criterion is considered conservative and pessimistic [10,12,15]. According to this, whatever alternative the decision-maker will choose, the nature will produce states that will minimize the decision's results. The decision-maker that uses this criterion is completely pessimist. This means that the decision-maker has a pessimistic attitude and behavior and tries to maximize the minimum possible result even in this pessimistic situations (to choose the best consequence from the bad ones).

If we consider the decision matrix from the Table 2 we can make the practical application of this criterion.

Therefore, according to this criterion, we note the optimal decision with:

$$D_0 = \max_i \min_j \{a_{ij}\},\ i = \overline{1,m},\ j = \overline{1,m}$$

$$D_0 = \max_i \min_j \{a_{ij}\} = \left\{ \begin{array}{l} A_1 = -2 \\ A_2 = 3 \\ A_3 = 6,5 \end{array} \right\} \Rightarrow A_3 = 6,5.$$

The first step of the decision-maker was to choose from the decisional matrix $\{a_{ij}\}$, that means, according to the pessimistic attitude, the lowest consequences, and the second step was to choose the best consequence from the minimum ones (maxi-mini), that means alternative $A_3 = 6.5$ **- program for acquiring new knowledge.**

This way corresponds to a decision-maker with an extreme pessimistic behavior.

Max-max criterion (optimistic)

This criterion assumes that the optimum decision variant is that for which we obtain the biggest advantages in the most favorable objective conditions.

In a situation of given decision, according to this criterion, the decision-maker with an optimistic attitude and behavior will have the concept that nature will produce states that will maximize the results of the decision [10,12,15]. Therefore, he assumes that the best state of nature appears and he consequently is going to select the best possible result. In order to elaborate the decision using this criterion, the decision-maker will first choose the maximum possible result for each alternative with the highest result. So, the optimal decision which we note with D_0 will be:

$$D_0 = \max_i \min_j \{a_{ij}\},\ i = \overline{1,m},\ j = \overline{1,m}$$

In our case:

$$D_0 = \max_i \min_j \{a_{ij}\} = \left\{ \begin{array}{l} A_1 = -15 \\ A_2 = 12 \\ A_3 = 6,5 \end{array} \right\} \Rightarrow A_3 = 15.$$

$A_1 = 15$ represents the best result among the alternatives from the matrix

This way corresponds to a decision-maker with an extreme optimist behavior.

Pessimistic-optimistic criterion (Hurwicz's criterion)

In this case, the decision-maker takes into consideration both the highest and the lowest possible results, weighted according to the decision-makers attitude (optimistic or pessimistic) that he has towards the decision. The weighing is made using a constant, also named the coefficient of the optimist ($0 \leq a \leq 1$).

Hurwicz's criterion suggests that every decision-maker is characterized *by a particular level of optimism*, marked with "a" (which normally is measured on a scale between 0 and 1), whose limits are:

- total pessimism", therefore "a" = 0;

- total optimism", therefore "a" = 1.

From here, logically, a *coefficient* of pessimism "1 - a" which is applied to the result, the worst consequence.

Hurwicz introduces a new value of assessing the alternatives "i" marked "Wi" that results from the sum of the result of the best alternative "i", "i" weighted with "a" and the result of the worst "i" weighted with "1 - a". Therefore:

$$W_i = \max_j a_{ij} + \min_j a_{ij} \times (1 - a) \text{ where } i = \overline{1, m},\ j = \overline{1, m}$$

and the optimal decision D_0 is:

$$D_0 = \max_i \{W_i\}$$

In our application, if we take "a" $= 0.7$ and considering the decision matrix from the Table 2, we will select the alternative based on the pessimistic-optimistic criterion, therefore:

Table 3. Decision matrix regarding the results of applying the pessimist-optimist criterion %

Nature states / Alternative	Maximum result	Minimum result	Expected value = a (maximum result) + (1 – a) (minimum result)
A_1	15	– 2	0.7 x 15 + (1 – 0.7) x (–2) = 9.9 → maximum
A_2	12	3	0.7 x 12 + (1 – 0.7) x 3 = 9.3
A_3	6.5	6.5	0.7 x 6.5 + (1 – 0.7) x 6.5 = 6.5

According to the optimistic-pessimistic criterion, the decision-maker will chose alternative A_1 (the training program), because it leads to the highest expected value of the results.

Minimum-maximum criterion or the criterion of minimizing the regret (Savage's criterion)

[10,12,15] According to this criterion, any rational decision-maker will try every time to minimize the biggest regret possible.

Using this criterion assumes:

- **first step**: construction of the matrix of regrets - In the matrix from table 2, the best results for each criterion $(S_1 S_2, S_3)$, thus: S_1- the best result is 15 S_2- the best result is 12 S_3 - the best result is 6.5.

- **second step**: from these results $(15; 12; 6.5)$, we will decrease the rest of the existent results in the initial matrix (Table 2), for obtaining the matrix of regrets, shown in Table 4, after which we will identify the best results for each criterion.

Table 4. Matrix of regrets in percentages

Nature states / Alternatives	Nature states			The highest regret
	S_1	S_2	S_3	
A_1	15-15=0	6.5-3=3.5	6.5-(-2)=4.5	4.5
A_2	15-12=3	6.5-6=0.5	6.5-3=3.5	**3.5** ←
A_3	15-6.5=8.5	6.5-6.5=0	6.5-6.5=0	8.5

• In the matrix above, for example:

- if the real state of nature that manifests is S_1 and the decision-maker chooses A_3, then he **will loose 8.5%** (15-6.5=8.5) of which he could get if he selected $A_1 \Rightarrow$ **he regrets**;

- if he selects A_2, then **we will loose** 3% (15-12=3) $\Rightarrow$ 3 regrets;

- if the decision-maker chooses A_1, then he will have no regrets (15-15=0) because he will obtain **the highest possible result** $\Rightarrow$ **15%.**

This reasoning is repeated for the other states of nature nature (S_2, S_3) and the results are presented in the matrix of regrets from the Table 4.

- *The minimum-maximum principle of regrets* - it is applied for identifying the biggest regret from each row (with this it is formed a new column) then we select among these the lowest regret. $A_1 \rightarrow 4,5$; $A_2 \rightarrow 3,5$; $A_3 \rightarrow 8,5$.

In the example from the matrix before the lowest regret is of 3.5% (minimum among the maximums). This criterion refers to the concept of regret, which is equivalent with the determination of losing the opportunity that means it indicates the significance of the suffered loss due to not selecting the best alternative.

Laplace criterion

The decision-maker that uses this criterion considers that all states of nature are equivalent probabilities,[10,15]. This means they have the same probability of apparition for each state of nature. The working method is similar with the one presented in the case of decisions adopted in conditions of risk. The equivalent probability is distributed to each state of nature, therefore, it is: $e = \frac{1}{S_j^2}$

Where: e = coefficient of equivalent probability S_j = number of nature states, $j = 1, ..., n$.

In the example of our matrix, considering that there can be three states of nature and that they have the same probability of apparition, the applying of Laplace criterion leads to the calculation of the best alternative of a probability of 0.33 (1:3), of realizing each state of nature. The obtained matrix is presented in Table 5.

Table 5. Matrix of the selection of the alternative based on the Laplace criterion

Nature states / Alternatives	Nature states			Result E(i)
	$S_1 = 0.33$	$S_2 = 0.33$	$S_3 = 0.33$	
A_1	0.33 x 15 = 4.95	0.33 x 3 = 0.99	0.33 x (-2) = -0.66	5,28
A_2	0.33 x 12 = 3.96	0.33 x 6 = 1.98	0.33 x 3 = 0.99	**6.93 →**
A_3	0.33 x 6.5 = 2.14	0.33 x 6.5 = 2.14	0.33 x 6.5 = 2.14	6.43

The calculation of the results is made by calculating the expected value E_i, that characterizes each alternative "i", and then from the E_i vector we will select the alternative with the maximum expected value. It is applied the known formula:

$$E_i = e\sum_{j=1}^{n} a_{ij} \text{ where } i = \overline{1,m},\ j = \overline{1,m}$$

and

$$D_0 = \max\{E_i\}$$

In our case, applying the data from the matrix from table 4 and the coefficient of equivalent probability (e = 0.33) we will obtain: $E_1 = 0.33[15+3+(-2)] = 5.28$ $E_2 = 0.33(12+6+3) = 6.93$ $E_3 = 0.33(6.5+6.5+6.5) = 6.43$ $D_0 = \max\{E_i\} = 6.93 = 6.93$

Therefore, the best alternative is A_2, with the highest expected result $\rightarrow 6.93$ - the program for developing skills.

This way is similar to the one in the unmeasured universe, in which the risk is equiprobable for each stage of nature.

The main drawback of this method is the fact that in reality is hard to believe that all the probabilities will be equal.

3 Conclusion

As we can observe in Table 5., the optimum strategy depends on the criterion chosen by the decision-maker. The choosing of the criterion by the decision-makers if justified by decision-makers' different nature (character, temperament, qualification etc.), by their different perception regarding the apparition probability of the future events and also, by their different attitude towards uncertainty. The decision-maker can be more optimist, more pessimist or he may to reduce the regret. Usually, the five criterions presented, although they lead to different alternative selections, in the made application, the optimum strategy is the same, that is - specializing programs - both in the case of the max-max criterion and the Hurwicz criterion. Also, the same optimum strategy - programs of abilities development - was selected both in the case of the Savage criterion and the Laplace criterion, while the Abraham Wald criterion was selected as optimum variant - program of acquiring new knowledge.

None of the criterions can be considered as being the best, each of these can have the same utility for different decision-makers as long as different persons often have common points of view over the solving of a problem.

The synthesis regarding the optimum strategy offers two alternatives for choosing the optimum decision, either specializing programs or programs of abilities development. Towards the two alternatives proposed by the results of the application, the final decision is the decision-maker's to make.

References

[1] Andron Daniela Roxana, *The knowledge- based organization: main characteristics and implications in risk management*, Rev.Academia Trupelor de Uscat, Sibiu, noiembrie 2007, pp. 82-91

[2] Andron Daniela Roxana, *Risk management in modern business organizations in the macroeconomic context of European integration*, Education and Academic Research Structures of the Knowledge-based Society, Universitatea "Alma Mater" Sibiu, 22-24 mai 2008, pp. 129-136 (in romanian)

[3] Andreica M., Stoica M., *Metode cantitative în management*, Editura Economică, Bucureşti, 1998(in romanian)

[4] Androniceanu Armenia, Burduş E.,*Managementul schimbării organizatorice*, Editura Economică, Bucureşti, 2008(in romanian)

[5] Burloiu P., *Managementul mondial al resurselor umane-o provocare*, Editura Economică, Bucureşti ,2010(in romanian)

[6] Ceauşu V., *De la incertitudine la decizie*, Editura Militară, Bucureşti, 1972(in romanian)

[7] Ciocoiu Carmen-Nadia, *Managementul riscului*, vol. I, Editura ASE, Bucureşti, 2008(in romanian)

[8] Ciocoiu Carmen-Nadia, *Managementul riscului*, vol. II, Editura ASE, Bucureşti, 2008(in romanian)

[9] Gorski Hortensia, *Managemen*t, Sibiu, Editura Universităţii Lucian Blaga, vol.I., 2008(in romanian)

[10] Ionescu Gh. Gh., Cazan E., Negruşa A. L., *Modelarea şi optimizarea deciziilor manageriale*, Editura Dacia, Cluj-Napoca, 1999(in romanian)

[11] Kissell R., Glantz M., *Optimal Trading Strategies -Quantitative Approaches for Managing Market Impact and Trading Risk*,Editura Amacom, Craiova, 2003

[12] Mărăcine, Virginia, *Decizii manageriale. Îmbunătăţirea performanţelor decizionale ale firmei*, Editura Economică, Bucureşti,1998(in romanian)

[13] Nica E., *Elaborarea şi folosirea studiilor de caz în managementul resurselor umane*, Editura Economică, Bucureşti, 2010(in romanian)

[14] Rotărescu E., *Alternative Selection under Risk Conditions in Human Resources Training and Development Through the Application of the Estimated Monetary Value and Decision Tree Analysis,* Rev. Academiei Forţelor Terestre, Sibiu, vol.XV, no.4(60), 2010, pp. 468-475

[15] Rusu Elisabeta, *Decizii optime în management, prin metode ale cercetării operaţionale: probleme şi studii de caz* Editura Enciclopedică Bucureşti, 2001(in romanian)

[16] Turban E., Meredith J., *Fundamentals of Management Science(5th Edition)*, Irwin, Boston, 1991

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